Mathematics Methods
Unit 4

Continuous random variable - Normal distribution

1. Normal distribution

Definition: Normal distribution (also known as the Gaussian distribution or the bell curve) is a continuous probability distribution wherein values lie in a symmetrical fashion mostly situated around the mean.

Examples of continuous random variable that are exactly or approximately normal:

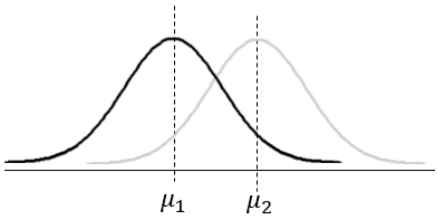
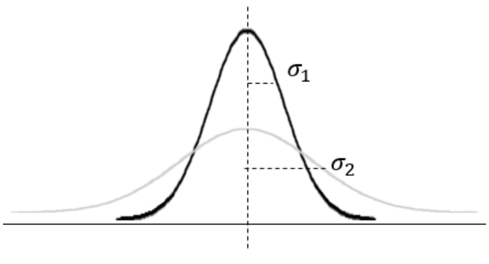
- Blood pressure
- Measurement error
- IQ scores
- Height

Probability density function of normal distribution

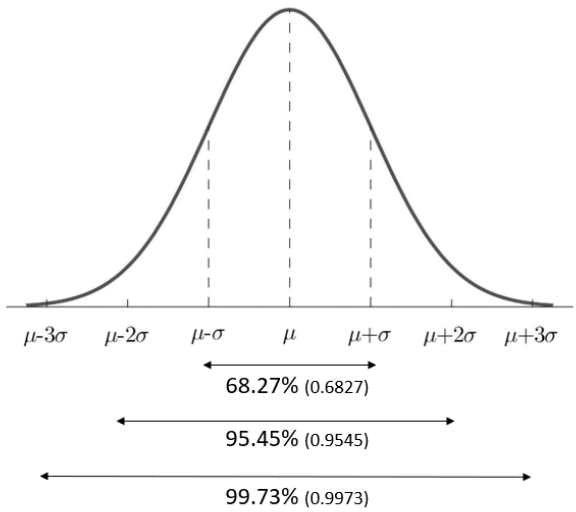
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}, -\infty < x < \infty$$

Basic properties of normal distribution:

- It is symmetric about the mean
- The mean = the mode = the median
- The curve is unimodal (one peak), maximum point at $(\mu, \frac{1}{\sigma\sqrt{2\pi}})$
- The curve approaches but never touches, the x-axis, as it extends farther and farther away from the mean $(-\infty < x < \infty)$
- Total area under the curve $= 1$ $(\int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}\, dx = 1)$
- Mean, μ and standard deviation, σ

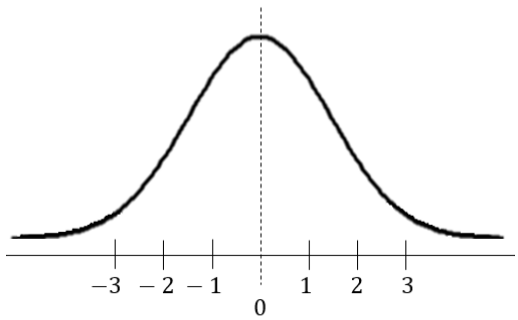
Mean, μ	The larger the mean, the greater the shift towards the right  $\mu_2 > \mu_1$
Standard deviation, σ	The larger the standard deviation, the greater the vertical compression  $\sigma_2 > \sigma_1$

Area under normal distribution and its corresponding standard deviation away from mean, μ.



2. Standard normal distribution

Definition: Standard normal distribution is a normal distribution with mean equals to 0 while standard deviation equals to 1.

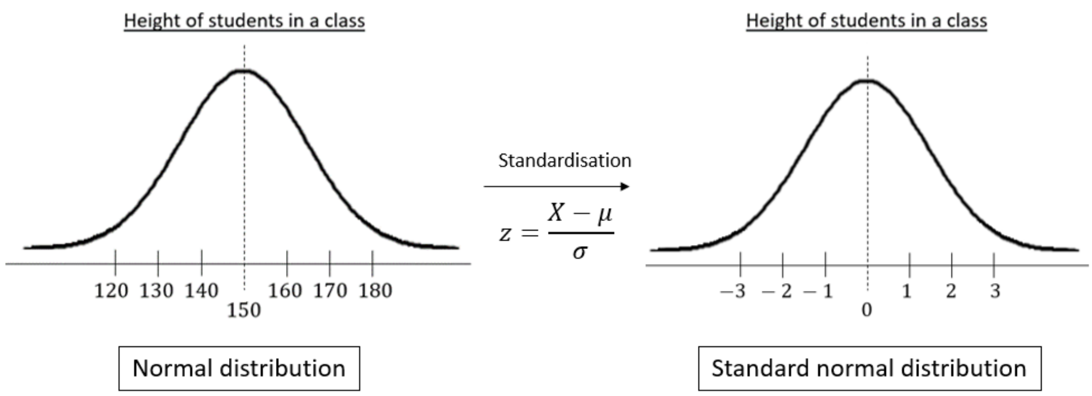


Standardisation formula:

$$z = \frac{X - \mu}{\sigma}$$

Basic properties of standard normal distribution: (same as normal distribution)
z score tells the number of standard deviation, σ is the value away from the mean, μ

Example:



3. Calculating the probability of normal distribution

(a) Given the X score

Example 1:
Find the probability of a test score less than 20% given that the test score is normally distributed $X \sim (50, 10^2)$.

$P(X < 50) = 0.00135$

Ti-nspire CX CAS guide
$norm\ Cdf(-\infty, 20{,}50{,}10)$

Example 2:
The length of screws in the toolbox is normally distributed with mean of 1 cm and standard deviation of 0.05 cm. find the probability that a randomly selected screw exceeds 1.1 cm.

$P(X > 1.1) = 0.02275$

Ti-nspire CX CAS guide
$norm\ Cdf(1.1, \infty, 1{,}0.05)$

(b) Given the percentage/ standard deviation away from mean

Example:
A set of normally distributed chisels has mean of 2 cm and standard deviation of 0.01 cm. Find the probability that a chisel picked at random is two standard deviation away from mean.

$$2\sigma = 2(0.01)$$
$$= 0.02\ cm$$

$$P(\mu - 2\sigma < X < \mu + 2\sigma) = P(2 - 0.02 < X < 2 + 0.02)$$
$$= (1.98 < X < 2.02)$$
$$= 0.9545$$

4. Finding the mean/ standard deviation/ X score

(a) Finding the mean and standard deviation

Example 1:
Given that X is a normal distribution that has mean, μ and variance of 20 as well that $P(X > 60) = 0.02235$ find the value of mean.

$$P(X > 60) = 0.02235$$
$$P\left(Z > \frac{60 - \mu}{20}\right) = 0.02235$$

Invnorm (1−0.02235, 0,1)
$$\frac{60 - \mu}{20} = 2.00747$$

$60 - \mu = 40.1494$
$-\mu = 40.1494 - 60$
$\mu = 19.85$

Example 2:
A random variable T has a normal distribution with mean of 39 and variance, σ^2. Given that $P(X > 42.5) = 0.098876$, find the standard deviation.

$P(X > 42.5) = 0.098876$
$P(Z > \frac{42.5 - 39}{\sigma}) = 0.098876$

Invnorm (1 – 0.098876, 0,1)
$\frac{42.5 - 39}{\sigma} = 1.28798$
$3.5 = 1.28798\sigma$
$\sigma = 2.71743$

Example 3:
The books in a library follows a normal distribution with mean, μ and variance 0.14. Given that $P(X < 20) = 0.0342$, find the mean.

$P(X < 20) = 0.0342$
$P(Z < \frac{20 - \mu}{0.004}) = 0.0342$

Invnorm (0.0342, 0,1)
$\frac{20 - \mu}{0.14} = -1.82236$
$\mu = 20.255$

Example 4:
Given that $X \sim N(\mu, \sigma^2)$, $P(10 < X < 20) = 0.234$ and $P(X < 10) = 0.0992$.

$P(X < 10) = 0.0992$
$P\left(Z < \frac{10 - \mu}{\sigma}\right) = 0.0992$

Invnorm (0.0992, 0,1)
$\frac{10 - \mu}{\sigma} = -1.28612$
$10 - \mu = -1.28612\sigma$ ....(1)

$P(X < 20) = 0.0992 + 0.234$
$= 0.3332$
$P\left(Z < \frac{20 - \mu}{\sigma}\right) = 0.3332$

Invnorm (0.3332, 0, 1)
$\frac{20 - \mu}{\sigma} = -0.431094$
$20 - \mu = -0.431094\sigma$ ....(2)

(1) – (2),
$-10 = -0.855026\sigma$
$\sigma = 11.7$

When $\sigma = 11.7$,
$10 - \mu = -1.28612(11.7)$
$\mu = 25.05$

END